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# COSMIC-RAY–INDUCED FILAMENTATION INSTABILITY IN COLLISIONLESS SHOCKS

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## ABSTRACT

We used unprecedentedly large 2D and 3D hybrid (kinetic ions – fluid electrons) simulations of non-relativistic collisionless strong shocks in order to investigate the effects of self-consistently accelerated ions on the overall shock dynamics. The current driven by suprathermal particles streaming ahead of the shock excites modes transverse to the background magnetic field. The Lorentz force induced by these self-amplified fields tends to excavate tubular, underdense, magnetic-field-depleted cavities that are advected with the fluid and perturb the shock surface, triggering downstream turbulent motions. These motions further amplify the magnetic field, up to factors of 50–100 in knot-like structures. Once downstream, the cavities tend to be filled by hot plasma plumes that compress and stretch the magnetic fields in elongated filaments; this effect is particularly evident if the shock propagates parallel to the background field. Highly-magnetized knots and filaments may provide explanations for the rapid X-ray variability observed in RX J1713.7-3946 and for the regular pattern of X-ray bright stripes detected in Tycho's supernova remnant.

*Keywords:* acceleration of particles — ISM: supernova remnants — magnetic fields — shock waves

## 1. INTRODUCTION

Following the pioneering idea of Fermi (1949), in the late '70s several authors realized that collisionless shocks are prominent sites for the acceleration of particles (Krymskii 1977; Axford et al. 1977; Bell 1978; Blandford & Ostriker 1978). More recently, the detection of narrow X-ray rims in young supernova remnants (SNRs), whose origin has been explained as synchrotron emission of relativistic electrons radiating in magnetic fields of a few hundreds $\mu$G, has provided evidence that the level of magnetization at SNR shocks is much larger than in the interstellar medium (see e.g., Vink & Laming 2003; Bamba et al. 2005; Parizot et al. 2006).

This association between particle acceleration and magnetic field amplification has been welcomed by theorists for several reasons. First, the super-Alfvénic streaming of accelerated particles is predicted to excite several different plasma instabilities (e.g., Bell 1978, 2004), which can account for the inferred levels of magnetization (at least as an extrapolation of the linear theory, see, e.g., Caprioli et al. 2009). Second, the interstellar turbulence cannot scatter particles efficiently enough to achieve the highest energies measured in Galactic cosmic rays, while self-generated magnetic fields can (e.g. Blasi et al. 2007). Finally, amplified magnetic fields may represent a key ingredient for explaining the steep spectra inferred from recent $\gamma$-ray observations (Caprioli 2012).

The details of such an interplay between accelerated particles and magnetic fields have not been completely understood yet, mostly because of the difficulty in accounting for the particle-wave coupling in the fully non-linear regime. Nevertheless, collisionless shocks are mediated by electromagnetic interactions only, so they can be modeled by iteratively moving particles on a grid according to the Lorentz force and adjusting the electromagnetic configuration via Maxwell equations. Such a particle-in-cell (PIC) approach provides great insight into the properties of collisionless shocks (see, e.g., Stroman et al. 2009; Ohira et al. 2009; Riquelme

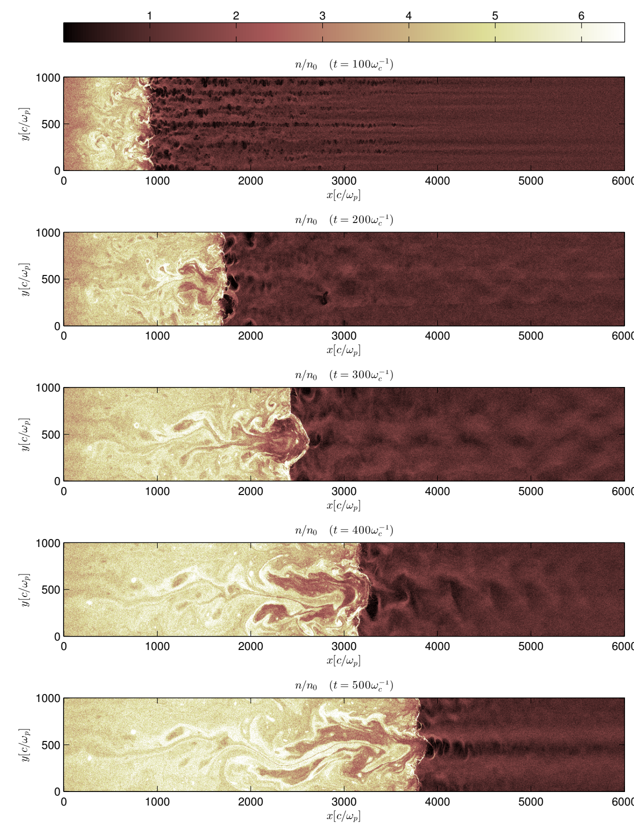

**Figure 1.** Evolution of the plasma density for a 2D, parallel collisionless shock with sonic and Alfvénic Mach numbers $M_s = M_A = 30$, as seen in the downstream frame. The total computational box measures $10^4 \times 10^3 (c/\omega_p)^2$, only a portion of which is showed here. For comparison, the nominal gyroradius of an ion with velocity $v_{sh}$ is $r_L = v_{sh}/\omega_c = 30c/\omega_p$ in the upstream magnetic field, while the most energetic particles at the end of the simulation have gyroradii as large as $r_L(E_{max}) \sim 300c/\omega_p$.

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& Spitkovsky 2009, 2010; Sironi & Spitkovsky 2011; Niemiec et al. 2012). However, PIC codes are computationally quite expensive, in that they require the plasma and gyration scales of both ions and electrons to be resolved. To partially mitigate this problem, one can adopt the so-called *hybrid* approach, in which the ions, which drive the dynamics, are treated kinetically while the (massless) electrons are treated as a neutralizing fluid. This approach neglects the small electron scales and allows the investigation of more macroscopic phenomena in much larger (in physical units) computational boxes. In particular, it has been widely exploited to study ion acceleration at collisionless shocks in different astrophysical environments (see, e.g., Winske 1985; Lipatov 2002; Giacalone et al. 1997; Giacalone & Ellison 2000; Giacalone 2004; Gargaté & Spitkovsky 2012).

Here we show the results of 2D and 3D hybrid simulations run with the non-relativistic code *dHybrid* (Gargaté et al. 2007). These simulations allow for unprecedentedly large computational boxes, especially in the direction transverse to the shock velocity, and can provide first-principle insight on the multi-dimensional structure of non-relativistic collisionless shocks. We focus our attention on shocks with high-Mach numbers ($M \gtrsim 30$); this regime is most relevant for SNR shocks, and so far has been scarcely studied because of the large dynamical range required to follow both thermal and non-thermal particles.

We investigate, for the first time in a self-consistent simulation of a collisionless shock with accelerated particles, the formation of peculiar structures in the upstream, which are shaped as tubular cavities surrounded by a net of dense filaments where the magnetic field is significantly amplified. That accelerated particles drive an instability that is filamentary in nature has already been put forward by Bell (2004, 2005); Zirakashvili et al. (2008), and Rogachevskii et al. (2012), who have run magnetohydrodynamical simulations with a fixed current imposed through a periodic box seeded with an initial turbulence.

Recently, Reville & Bell (2012) have provided an analytical derivation of the cavity growth rate, supporting their findings with simulations that couple a fluid treatment of the background plasma with a kinetic description of the energetic ions driving the instability. The instability was studied in a 2D slab geometry representing a section of the upstream perpendicular to the shock normal.

The simulations presented here differ from those in the literature in several respects: 1) the ion current is time-dependent and self-consistently generated by shock acceleration, and not estimated by assuming the relativistic ions to be isotropic in the shock frame, which translates into an anisotropy of order $\sim v_s/c$ in the upstream reference frame (here $v_s$ is the shock speed); 2) we do not need to seed the fluid with pre-existing turbulence, since accelerated particles generate it by themselves via streaming instability; 3) the more physical setup where filamentation develops while the fluid is advected towards the shock allows us to properly study the evolution and the spatial features of cavities and filaments; 4) simulating the global shock structure allows us to show how, when advected through the shock, cavities and filaments affect the nature of the discontinuity and the magnetic field topology.

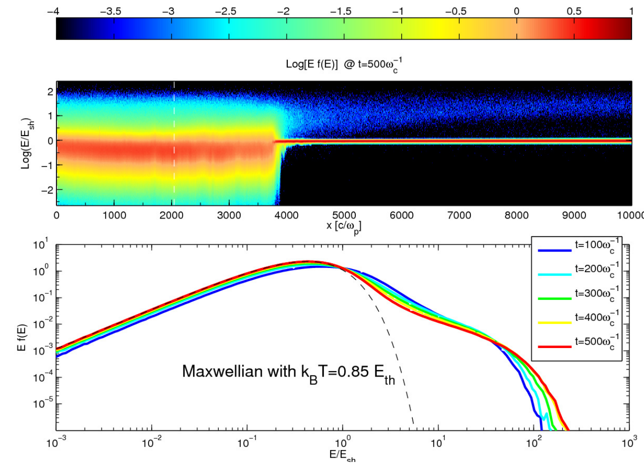


**Figure 2.** *Top panel:* Particle energy spectrum $f(E)$ as a function of $x$ in units of $E_{sh} = mv_{sh}^2/2$ at time $t = 500\omega_c^{-1}$. *Bottom panel:* Time evolution of the particle spectrum for $x < 2000c/\omega_p$, as seen in the downstream reference frame. For comparison, the Maxwellian distribution corresponding to a temperature $T = 0.85E_{th}/k_B$ is showed as well (dashed line), with $E_{th} = 3/8E_{sh}$ the temperature given by standard jump conditions.

In §2 we outline the main features of our 2D and 3D simulations for both parallel and oblique shocks, and discuss the mechanisms that lead to the formation of cavities and filaments. In §3 we discuss some possible observational implications of such filamentation in the context of the synchrotron emission detected in young SNRs like Tycho and RX J1713.7-3946.

## 2. HYBRID SIMULATIONS

Most of the simulations presented here are for a 2D parallel shock (i.e., background field $\mathbf{B}_0$ aligned with the shock velocity $\mathbf{v}_{sh} = v_{sh}\hat{x}$) with sonic and Alfvénic Mach numbers $M_s = v_{sh}/c_s = M_A = v_{sh}/v_A \equiv M = 30$ in the downstream (simulation) reference frame, corresponding to $M \approx 40$ in the upstream one. In the 2D simulations, we include both in-plane and out-of-plane components of the ion momenta and electromagnetic fields.

In our non-relativistic framework, velocities are normalized to the Alfvén speed $v_A$, lengths to the ion skin depth $c/\omega_p = v_A/\omega_c$, and times to $\omega_c^{-1}$, with $\omega_p = \sqrt{4\pi n_0 e^2/m}$ and $\omega_c = eB_0/mc$ being the ion plasma and cyclotron frequencies ($m$ is the proton mass). Density and magnetic fields are measured in units of the upstream initial values, $n_0$ and $B_0$. The ion skin depth is resolved with two grid cells and the computational box measures $(L_x \times L_y) = 10^4 \times 10^3 (c/\omega_p)^2$. The time step is chosen as $\Delta t = 0.001\omega_c^{-1}$ to enhance the energy conservation with a very small Courant number. The shock is generated by introducing a perfectly reflecting wall (left boundary in the figures) that initially produces counter-streaming particles. This configuration is unstable and the system promptly forms a propagating sharp discontinuity with width of a few gyroradii of the downstream thermal particles, behind which the cold upstream flow is isotropized.

The global evolution of the shock obtained with the present setup is showed in Figure 1, where the density is plotted at different times. The shock propagates to the right along $x$ axis. Since $M \gg 1$, the ratio between the plasma density behind and ahead of the shock rapidly

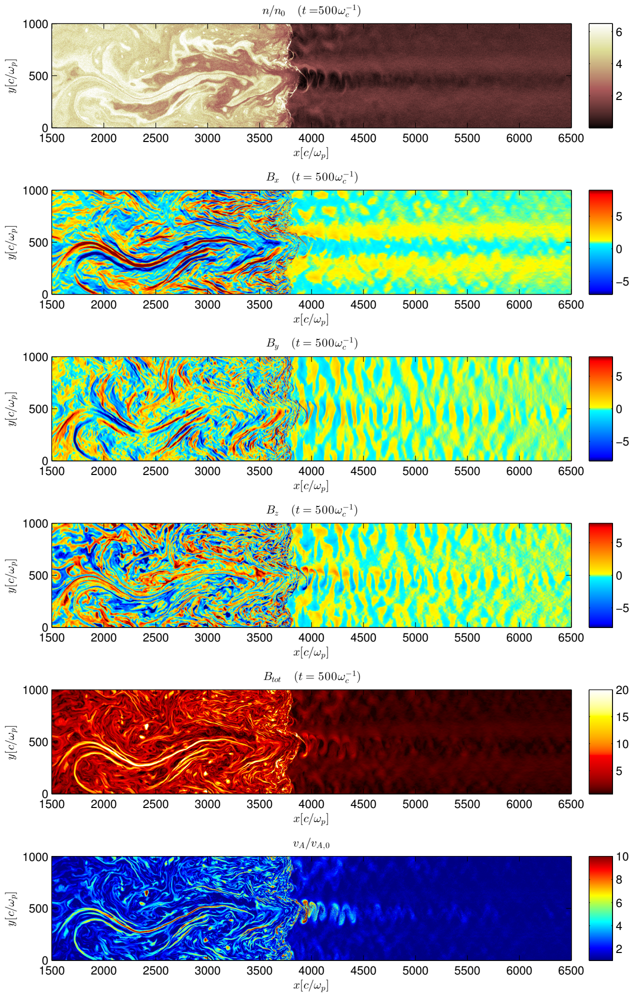


**Figure 3.** Density, parallel ($B_x$), transverse ($B_y$,$B_z$) and total ($B_{tot}$) magnetic field, and Alfvén velocity $v_A = B_{tot}/\sqrt{4\pi mn}$ for the 2D simulation in Figure 1 at $t = 500\omega_c^{-1}$. All the quantities are normalized to their initial values. A filament with $B_{tot} \approx 15{-}20B_0$ and some knots where $B_{tot} \approx 20 - 40B_0$ are clearly visible.

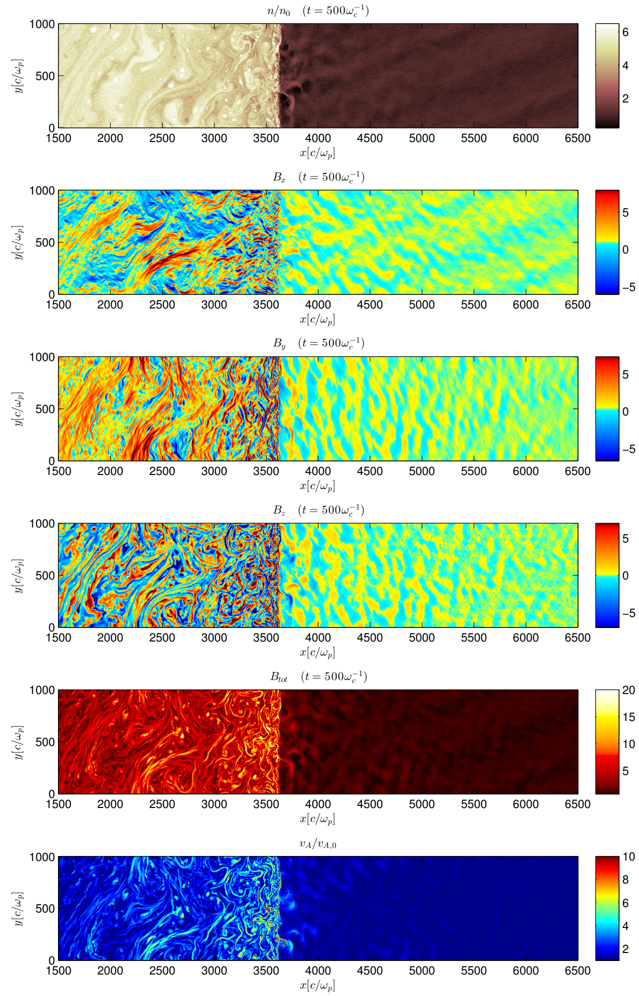


**Figure 4.** As in Figure 3, but for an oblique shock with $\mathbf{B}_0$ in the $xy$-plane, at an angle $\vartheta = 20°$ with respect to $\mathbf{v}_{sh}$.

approaches 4, and the post-shock fluid comes to rest in the wall (downstream) frame, dissipating the inflowing kinetic energy into heat.

Figure 2 shows a snapshot of the ion distribution function at $t = 500\omega_c^{-1}$ and the time evolution of the downstream spectrum. According to first-order Fermi mechanism, particles scattered back and forth across the shock get accelerated effectively: the ion energy spectrum quickly develops a non-thermal power-law tail that, for the chosen parameters, comprises $\sim 15\%$ of the total ion energy. The details of the ion energy spectrum will be discussed in a forthcoming paper.

The peak of the ion Maxwellian distribution is consistently shifted to lower temperatures by about the same amount with respect to the pure gaseous case (i.e., the case with no accelerated particles). At the same time, since all the particles are coupled through electromagnetic interactions, the pressure in non-thermal ions propagating upstream slows down the incoming fluid producing a so-called precursor (top panel in Figure 2), a distinctive feature of shocks modified by the back-reaction of accelerated particles (see, e.g., Jones & Ellison 1991; Malkov & O'C. Drury 2001, for reviews).

## 2.1. *Filamentation instability*

The most noticeable feature in Figure 1 is the formation of low-density cavities ahead of the shock ($n/n_0 \lesssim 10^{-2}$), whose origin can be explained in the following way. Particles accelerated at the shock propagate upstream against the incoming fluid (top panel of Figure 2). At first, the magnetic field is the background $\mathbf{B_0}$ only; therefore, ions do not diffuse in pitch angle, giving rise to a coherent current $\mathbf{J} \parallel \mathbf{B_0}$. However, the super-Alfvénic streaming of the accelerated particles tends to excite transverse magnetic modes via plasma instabilities, eventually seeding the upstream medium with a transverse $\delta\mathbf{B}$. The resulting Lorentz force $\mathbf{F}_L \propto -\mathbf{J} \times \delta\mathbf{B}$ pushes the plasma (and the frozen-in magnetic field) away from the region where the current is stronger, also focusing the energetic particles and helping to sustain the instability (Bell 2005). The net result is the formation of low-density tunnels filled with supra-thermal ions, modulated by the period of the underlying magnetic perturbation (see, e.g., the snapshot at $t = 300\omega_c^{-1}$ in Figure 1).

A complete discussion of the most unstable modes excited in large Mach number parallel shocks is beyond the scope of this paper. The reader may refer to Amato & Blasi (2009) for a kinetic description of resonant and non-resonants modes excited in the linear regime and to

the simulations by, e.g., Stroman et al. (2009); Riquelme & Spitkovsky (2009); Gargaté et al. (2010) for a study of the fully non-linear regime. Note that Bell's non-resonant modes are not particularly relevant here since their growth rate is typically shorter than $\sim \omega_c^{-1}$, and on the advection time-scale $\approx L_{box}/v_{sh} \gg \omega_c^{-1}$ such small-wavelength modes have already reached saturation, while modes resonant with the accelerated particles are still growing (see, e.g., Gargaté et al. 2010). A sizable fraction of the energy in the magnetic turbulence is stored on scales as large as 10–100$c/\omega_p$, comparable with, or larger than, the gyroradius $\sim v_{sh}/\omega_c = 30c/\omega_p$ of ions with velocity $\sim v_{sh}$.

### 2.2. *Growth of cavities and filaments*

An interesting question is what determines the size of the cavities. Reville & Bell (2012) argued that the growth of the cavities is suppressed when their size becomes comparable with the gyroradius of the ions carrying most of the current, since the scattering of these particles would suppress the instability. In our setup, the growth rate of the filamentation may also be limited by advection, while the current may increase with time because the acceleration becomes more and more efficient.

In our simulations cavities grow while being advected toward the shock and, by comparing different panels of Figure 1, we notice that the typical transverse size of the cavities impacting the shock increases with time. This is qualitatively consistent with a cavity growth rate $\Gamma \propto |B_\perp|\xi_{acc}^{1/2}$, where $|B_\perp|$ is the averaged magnitude of transverse component of $\mathbf{B}$, and $\xi_{acc}$ is the fraction of the total pressure in accelerated particles (Reville & Bell 2012). In our simulations, $\xi_{acc}$ and, in turn, $|B_\perp|$ increase with time (see the non-thermal tail in the spectra of Figure 2). At $t \gtrsim 400\omega_c^{-1}$ the transverse size of the biggest cavity measures $\approx 300c/\omega_p$, and it is comparable with the gyroradius of the ion with the highest energy, $E_{max} \sim 100E_{sh}$, with $E_{sh} = mv_{sh}^2/2$ (Figure 2).

Lower-energy particles resonate with the self-generated magnetic turbulence and are effectively scattered, as shown in the top panel of Figure 2. The density of ions with $E \lesssim 10E_{sh}$ drops with the distance from the shock: if these particles were not deflected back, they would escape the box on a timescale shorter than the simulation one. By undergoing multiple interactions with the shock, ions are efficiently accelerated, so that $E_{max}(t)$ increases with time. On the other hand, freshly accelerated ions with energies $\sim E_{max}$ do not have enough waves to resonate with and stream more freely, contributing the most to the current because of their anisotropy.

For $t > 500\omega_c^{-1}$, $E_{max}(t)$ no longer increases due to the finite size of the box in the $x$ direction: however, we checked that this limitation is removed for larger boxes and consistently longer simulation times. Enlarging the box in the $y$ direction, instead, does not accommodate larger and larger cavities, attesting to the physical origin of their diameter. The transverse size of the box must, however, be large enough to resolve the gyration of the most energetic ions at any time: the smaller the box in the $y$ direction, the earlier filamentation is suppressed. Previous simulations (e.g., Giacalone 2004; Gargaté & Spitkovsky 2012) used smaller boxes, and, therefore, the effect was less evident.

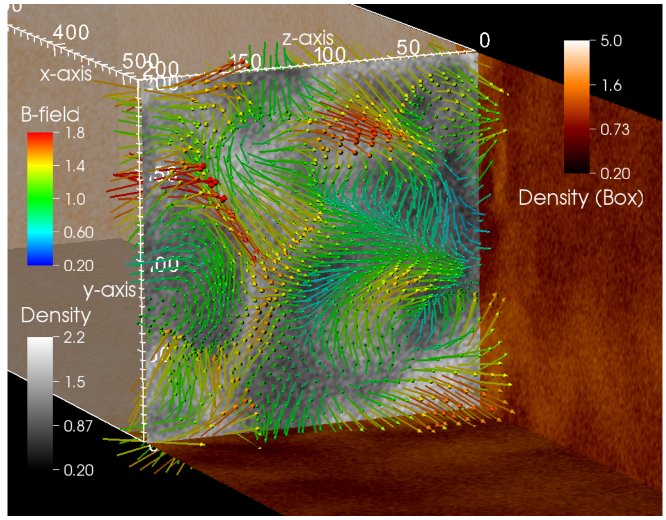


**Figure 5.** Snapshot at $t = 175\omega_c^{-1}$ of a 3D hybrid simulation of a parallel shock with $M_s = M_A = 6$ in a $2000 \times 200 \times 200(c/\omega_p)^3$ box (see text for further details). The color code (right colorbar) on the box sides shows the particle density in units of $n_0$, while the vertical slice illustrates a section of the fluid at $x = 520c/\omega_p$, i.e., immediately ahead of the shock. In the slice, the grey-scale code corresponds to the ion density, while the colored vectors show strength and direction of the magnetic field, in units of $B_0$. Notice the correlation between underdensity and low $B$-field and how the magnetic field is mainly along $\hat{x}$ in the filaments and coiled inside the cavities.

### 2.3. *Magnetic field structure*

In Figure 3 the parallel ($B_x$), transverse ($B_y$,$B_z$) and total ($B_{tot}$) magnetic fields are shown for the 2D case at $t = 500\omega_c^{-1}$. In particular, the filamentary structure of the upstream is very evident in $B_x$, where organized lateral modulations significantly affect the mean magnetic field topology. While at parallel shocks the ion-driven streaming instability is expected to produce a perpendicular component of the field, a change in $B_x$ is a non-linear effect which is not predicted in the quasi-linear analysis. Also, $B_x$ tends to be evacuated from cavities and pushed into filaments, while the transverse component winds up around the cavity itself (Figure 3).

There is also another interesting topological effect that arises in the scenario presented here. When a cavity is advected through the shock, a bubble of relatively rarefied and cold (it contains less kinetic energy to be dissipated) plasma is produced in the downstream (see, e.g., Figure 1 at $t = 300\omega_c^{-1}$). Such a configuration is Rayleigh–Taylor unstable, and each bubble is promptly filled by a *plume* of hotter and denser plasma, as visible in Figure 1 between $t = 400$ and $500\omega_c^{-1}$. These macroscopic motions systematically stretch the magnetic field along the cavity axis, as can be seen in $B_x$ and $B_{tot}$ in Figure 3.

A similar phenomenology is recovered for oblique shocks, too. Figure 4 shows the output of a simulation for a shock with an angle of 20° between $\mathbf{B}_0$ and $\mathbf{v}_{sh}$, and all the other parameters fixed as in the parallel case. Cavities always develop along the direction of the upstream background field, being driven by energetic particles spiraling around $\mathbf{B}_0$. However, in this case the shock is less corrugated, and downstream cavities are squeezed more rapidly, preventing the formation of very long filaments.

### 2.4. *3D simulations*

To better understand the topology of these structures, we also ran 3D simulations with *dHybrid*. Both the shock velocity and the computational domain are scaled down by a factor of 5 with respect to the 2D case to compensate for the higher computational effort without changing the ratio of the box size and the gyroradius of typical particles, $r_L \propto v_{sh}$. More precisely, we set $M_s = M_A = 6$ in a $2000\times 200 \times 200 (c/\omega_p)^3$ box, allowing for the same time and space resolution as in the 2D case.

Figure 5 shows the rendering of the density and field structure near the shock in a 3D simulation. The slice shows a transverse section of the fluid immediately ahead of the shock, where the contrast between filaments and cavities may be as large as 5–10 in both the ion density (grayscale) and in total magnetic field (color-coded vectors). There is a clear correlation between regions of enhanced magnetic field and enhanced density, confirming the formation of rarefied, low-$B$ cavities surrounded by regions of denser plasma permeated by a stronger field.

An interesting feature clearly visible in 3D simulations is that the field is mainly parallel to the shock normal in the dense filaments, while inside the cavities the field is coiled and develops a transverse component comparable with, and even larger than the one along $\hat{x}$ (see also Reville & Bell 2012).

## 3. OBSERVATIONAL CONSEQUENCES

Very generally, we can assess that filamentation provides substantial magnetic field amplification even in the pre-shock medium. The magnetic-field strength in some filaments may become as large as $\sim 5B_0$, while cavities may become effectively demagnetized. While low-energy accelerated ions are focused in the inner regions of the cavities, ions with gyroradii comparable with the cavity transverse size can feel the effect of the field amplification and scatter more efficiently. In particular, the increase of $B$ and the decrease of $n$ in the cavities lead to a local Alfvén velocity significantly larger than the initial one (bottom panels of Figures 3 and 4). This suggests that the phase velocity of the magnetic perturbations may become a non-negligible fraction of the fluid velocity even for strong shocks, eventually affecting the scattering and, in turn, the spectra of the accelerated particles. In particular, the effect may be crucial to explain the steep inferred from $\gamma$-ray observations of SNRs (Caprioli 2012).

Also, the thermal plasma is affected by the filamentation process: ahead of the shock the temperature becomes several times larger than at the beginning of the simulation. Quite interestingly, the pressure in the thermal plasma and in the magnetic turbulence are almost in equipartition along the precursor.

Due to the non-linear processes developing in the upstream, the magnetic field orientation and the local Alfvénic and sonic Mach numbers may vary significantly immediately ahead of the shock (Figures 3 and 4); different patches of the upstream plasma are therefore shocked in different ways, eventually leading to the onset of coherent motions that stretch the field preferentially along the cavities, but also turbulent motions that effectively stir the downstream fluid on smaller scales.

The net result is that the post-shock magnetic field may become even larger than what is expected from a simple compression by a factor $\sim 4$ of the transverse component of the pre-shock field. In Figure 3, in addition to an elongated filament where $B \geq 15 - 20B_0$, it is possible to spot knots with $B \sim 30 - 40B_0$. Runs with larger Mach numbers ($M = 50$) show that the total $B$-field in knot-like structures can easily reach up to 90–$100B_0$. These structures may resemble the ones detected in RX J1713.7-3946, where the fields inferred by the X-ray variability are as strong as 1mG (Uchiyama et al. 2007), namely a few hundred times stronger than the typical interstellar magnetic field.

In application to SNRs, if the cavity sizes continue to grow to the gyroradius of ions with $E_{max}$, filamentation instability might also account for the detection of a characteristic pattern of X-ray-bright stripes (Eriksen et al. 2011) in the South-East region of Tycho (see also Bykov et al. 2011). In this case, stripes would develop where the shock normal is parallel to the large-scale magnetic field, and the stripe spacing would correspond to the gyroradius of protons with energy $\sim 10^6$GeV, a value compatible with the maximum energy achieved in Tycho as inferred from $\gamma$-ray data (Morlino & Caprioli 2012).

Though the hybrid simulations presented here cannot properly reproduce the large physical scales relevant for real SNRs, they qualitatively confirm that the instabilities driven by the accelerated particles can lead to the formation of filaments and knots sharing some properties — such as the geometry and the enhancement of the magnetic field — consistent with several observational signatures detected in young SNRs.

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## REFERENCES

Amato, E., & Blasi, P. 2009, MNRAS, 392, 1591
Axford, W. I., Leer, E., & Skadron, G. 1977, in International Cosmic Ray Conference, Vol. 2, *Acceleration of Cosmic Rays at Shock Fronts*, 273–+
Bamba, A., Yamazaki, R., Yoshida, T., Terasawa, T., & Koyama, K. 2005, ApJ, 621, 793
Bell, A. R. 1978, MNRAS, 182, 147
—. 2004, MNRAS, 353, 550
—. 2005, MNRAS, 358, 181
Blandford, R. D., & Ostriker, J. P. 1978, ApJL, 221, L29
Blasi, P., Amato, E., & Caprioli, D. 2007, MNRAS, 375, 1471
Bykov, A. M., Ellison, D. C., Osipov, S. M., Pavlov, G. G., & Uvarov, Y. A. 2011, ApJ, 735, L40
Caprioli, D. 2012, JCAP, 7, 38
Caprioli, D., Blasi, P., Amato, E., & Vietri, M. 2009, MNRAS, 395, 895
Eriksen, K. A., Hughes, J. P., Badenes, C., et al. 2011, ApJl, 728, L28+
Fermi, E. 1949, Phys. Rev., 75, 1169

Gargaté, L., Fonseca, R. A., Niemiec, J., et al. 2010, ApJl, 711, L127
Gargaté, L., & Spitkovsky, A. 2012, ApJ, 744, 67
Gargaté et al., L. 2007, Computer Physics Communications, 176, 419
Giacalone, J. 2004, ApJ, 609, 452
Giacalone, J., Burgess, D., Schwartz, S. J., Ellison, D. C., & Bennett, L. 1997, J. Geophys. Res., 102, 19789
Giacalone, J., & Ellison, D. C. 2000, J. Geophys. Res., 105, 12541
Jones, F. C., & Ellison, D. C. 1991, Space Science Reviews, 58, 259
Krymskii, G. F. 1977, Akademiia Nauk SSSR Doklady, 234, 1306
Lipatov, A. S. 2002, The hybrid multiscale simulation technology: an introduction with application to astrophysical and laboratory plasmas, Scientific computation (Berlin; New York: Springer)
Malkov, M. A., & O'C. Drury, L. 2001, Reports of Progress in Physics, 64, 429
Morlino, G., & Caprioli, D. 2012, A&A, 538, A81
Niemiec, J., Pohl, M., Bret, A., & Wieland, V. 2012, ArXiv e-prints
Ohira, Y., Reville, B., Kirk, J. G., & Takahara, F. 2009, ApJ, 698, 445
Parizot et al., E. 2006, A&A, 453, 387
Reville, B., & Bell, A. R. 2012, MNRAS, 419, 2433
Riquelme, M. A., & Spitkovsky, A. 2009, ApJ, 694, 626
—. 2010, ApJ, 717, 1054
Rogachevskii, I., Kleeorin, N., Brandenburg, A., & Eichler, D. 2012, ApJ, 753, 6
Sironi, L., & Spitkovsky, A. 2011, ApJ, 726, 75
Stroman, T., Pohl, M., & Niemiec, J. 2009, ApJ, 706, 38
Uchiyama, Y., Aharonian, F. A., Tanaka, T., Takahashi, T., & Maeda, Y. 2007, Nature, 449, 576
Vink, J., & Laming, J. M. 2003, Ap. J., 584, 758
Winske, D. 1985, Space Sci. Rev., 42, 53
Zirakashvili, V. N., Ptuskin, V. S., & Völk, H. J. 2008, ApJ, 678, 255